INDIAN SCHOOL AL WADI AL KABIR

Assessment - 1

Class: XII — Sub: MATHEMATICS (041) — Max Marks: 80

Date: 22.09.2024 — Set - II -ANSWER KEY — Time: 3 hr

1	b) $\frac{x^2}{2}$ + log |x| + c
2	c) k = 1
3	c) (2, ∞)
4	b) $\frac{3}{4t}$
5	d) - $\frac{\pi}{7}$
6	c) 25y
7	c) discontinuous at exactly two points
8	d) -3
9	b) $\frac{1}{4}$
10	d) -75
11	a) 8 & 8
12	d) A + B = 0
13	c) tan x - cot x + C
14	c) 1.4 π cm/s
15	d) 512
16	b) e^x sec x + C
17	a) π
18	a) BA^{-1}
19	(A) Both A and R are true and R is the correct explanation of A
20	(C) A is true but R is false
21	Determinant = $1(2x^2 + 4) - 2(-4x - 20) = 86$ $= x^2 + 4x - 21 = 0$ $x = 3, -7$

22

$$\text{Let } I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{dx}{1+\sqrt{\tan x}} = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\cos x}\,dx}{\sqrt{\cos x}+\sqrt{\sin x}} \quad \ldots (1)$$

$$I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\cos\left(\frac{\pi}{3}+\frac{\pi}{6}-x\right)}\,dx}{\sqrt{\cos\left(\frac{\pi}{3}+\frac{\pi}{6}-x\right)}+\sqrt{\sin\left(\frac{\pi}{3}+\frac{\pi}{6}-x\right)}}$$

$$= \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}\,dx \quad \ldots (2)$$

Adding (1) and (2),

$$2I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} dx = [x]_{\frac{\pi}{6}}^{\frac{\pi}{3}} = \frac{\pi}{3} - \frac{\pi}{6} = \frac{\pi}{6}$$

$$I = \frac{\pi}{12}$$

-OR-

$$\text{Let } I = \int_1^3 \frac{\sqrt[3]{x}}{\sqrt[3]{x}+\sqrt[3]{4-x}}\,dx \ \ldots\ldots (1)$$

$$I = \int_1^3 \frac{\sqrt[3]{4-x}}{\sqrt[3]{4-x}+\sqrt[3]{4-(4-x)}}\,dx$$

$$I = \int_1^3 \frac{\sqrt[3]{4-x}}{\sqrt[3]{4-x}+\sqrt[3]{x}}\,dx \ \ldots\ldots (2)$$

Adding (1) and (2), we get

$$2I = \int_1^3 \frac{\sqrt[3]{x}+\sqrt[3]{4-x}}{\sqrt[3]{4-x}+\sqrt[3]{x}}\,dx$$

$$2I = \int_1^3 1\,dx$$

$$2I = x|_1^3$$

$2I = 3 - 1 = 2$ gives $I = 1$

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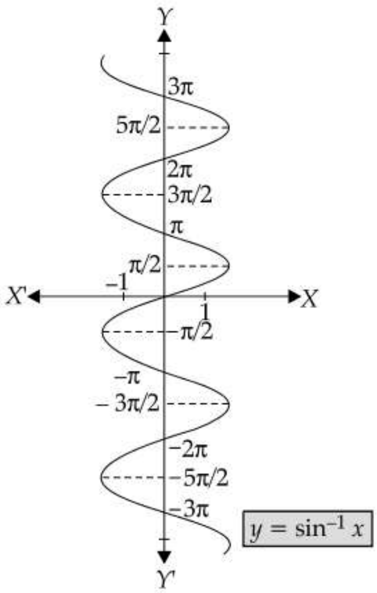

- OR -

$$\sin^2\left(\cos^{-1}\frac{1}{4}\right) + \cos^2\left(\sin^{-1}\frac{1}{3}\right)$$

$$= 1 - \cos^2\left(\cos^{-1}\frac{1}{4}\right) + 1 - \sin^2\left(\sin^{-1}\frac{1}{3}\right)$$

$$= 1 - \left(\frac{1}{4}\right)^2 + 1 - \left(\frac{1}{3}\right)^2$$

$$= 2 - \frac{1}{16} - \frac{1}{9}$$

$$= 2 - \frac{25}{144} = \frac{288-25}{144} = \frac{263}{144}$$

24

Given function $f(x) = \cos x, \forall x \in \mathbb{R}$

$$f\left(\frac{\pi}{2}\right) = \cos\frac{\pi}{2} = 0 \qquad \Rightarrow f\left(\frac{-\pi}{2}\right) = \cos\frac{\pi}{2} = 0 \qquad \Rightarrow f\left(\frac{\pi}{2}\right) = f\left(\frac{-\pi}{2}\right)$$

But $\frac{\pi}{2} \notin \frac{-\pi}{2} = 0$

So, $f(x)$ is not one-one

Now, $f(x) = \cos x, \forall \in \mathbb{R}$ is not onto as there is no preimage for an real numbe

Which does not being to the intervals, $[-1, 1]$, the range of $\cos x$.

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Given that the function is continuous at $x = 2$.

Therefore, LHL = RHL = $f(2)$

$$\Rightarrow \lim_{x\to 2^-} f(x) = \lim_{x\to 2^+} f(x) = f(2)$$

$$\Rightarrow \lim_{x\to 2^-} 5 = \lim_{x\to 2^+} ax + b = 5$$

$$\Rightarrow 2a + b = 5 \quad \ldots\text{(i)}$$

Given that the function is continuous at $x = 10$.

Therefore, LHL = RHL = $f(10)$

$$\Rightarrow \lim_{x\to 10^-} f(x) = \lim_{x\to 10^+} f(x) = f(10)$$

$$\Rightarrow \lim_{x\to 10^-} ax + b = \lim_{x\to 10^+} 21 = 21$$

$$\Rightarrow 10a + b = 21 \quad \ldots\text{(ii)}$$

Solving the equation (i) and (ii), we get $a = 2$ $b = 1$

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here, $x = a\left(\cos t + \log\tan\frac{t}{2}\right)$

Therefore, $\frac{dx}{dt} = a\left(-\sin t + \frac{1}{\tan\frac{t}{2}}.\sec^2\frac{t}{2}.\frac{1}{2}\right) = a\left(-\sin t + \frac{\cos\frac{t}{2}}{\sin\frac{t}{2}}.\frac{1}{\cos^2\frac{t}{2}}.\frac{1}{2}\right)$

$$= a\left(-\sin t + \frac{1}{2\sin\frac{t}{2}\cos\frac{t}{2}}\right) \qquad = a\left(-\sin t + \frac{1}{\sin t}\right)$$

$$= a\left(\frac{-\sin^2 t+1}{\sin t}\right) \qquad = a\left(\frac{\cos^2 t}{\sin t}\right)$$

$y = a\sin t \qquad \frac{dy}{dt} = a\cos t$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{a\cos t}{a\left(\frac{\cos^2 t}{\sin t}\right)} = \frac{\sin t}{\cos t} = \tan t$$

27

Let $I = \int 1\cdot\tan^{-1}x\,dx$

$I = \tan^{-1}x\int 1\,dx - \int\left\{\left(\frac{d}{dx}\tan^{-1}x\right)\int 1\cdot dx\right\}dx$

$= \tan^{-1}x\cdot x - \int\frac{1}{1+x^2}\cdot x\,dx$

$= x\tan^{-1}x - \frac{1}{2}\int\frac{2x}{1+x^2}dx$

$= x\tan^{-1}x - \frac{1}{2}\log|1+x^2| + C$

$= x\tan^{-1}x - \frac{1}{2}\log(1+x^2) + C$

$\tan^{-1}x = x\tan^{-1}x - \frac{1}{2}\log(1+x^2) + C$

-OR-

Consider $I = \int_1^3 |x^2-2x|\,dx$

$$|x^2-2x| = \begin{cases} -(x^2-2x) & \text{where } 1\le x<2 \\ (x^2-2x) & \text{where } 2\le x\le 3\end{cases}$$

$I = \int_1^2 |x^2-2x|\,dx + \int_2^3 |x^2-2x|\,dx$

$I = \int_1^2 -(x^2-2x)\,dx + \int_2^3 (x^2-2x)\,dx$

$I = -\left[\frac{x^3}{3}-x^2\right]_1^2 + \left[\frac{x^3}{3}-x^2\right]_2^3$

$I = -\left[\frac{8}{3}-4-\frac{1}{3}+1\right]+\left[9-9-\frac{8}{3}+4\right]$

$= -\frac{7}{3}+3-\frac{8}{3}+4$

$I = 7-5 = 2$

28

Let x be the length of a side, V be the volume and S be the surface area of the cube.
$V = x^3$ and $S = 6x^2$, where x is a function of time t.

$\frac{dV}{dt} = 9\text{cm}^3/\text{s}$ (Given)

$9 = \frac{dV}{dt} = \frac{d}{dt}(x^3) \quad = \frac{d}{dx}(x^3)\cdot\frac{dx}{dt}$

$= 3x^2\cdot\frac{dx}{dt}$

$\frac{dx}{dt} = \frac{3}{x^2}$

$\frac{dS}{dt} = \frac{d}{dt}(6x^2) \quad = \frac{d}{dx}(6x^2)\cdot\frac{dx}{dt}$

$= 12x\cdot\left(\frac{3}{x^2}\right) = \frac{36}{x}$

when $x = 10$ cm,

$\frac{dS}{dt} = 3.6\text{ cm}^2/\text{s}$

- OR -

We have $y = \frac{2}{3}x^3+1 \quad \ldots(i)$

$\Rightarrow \frac{dy}{dt} = \frac{2}{3}\times 3x^2\times\frac{dx}{dt} = 2x^2\frac{dx}{dt}$

$\because \frac{dy}{dt} = 2\frac{dx}{dt}$

$\Rightarrow 2\frac{dx}{dt} = 2x^2\frac{dx}{dt}$

$\Rightarrow x^2 = 1 \qquad \therefore x = \pm 1$

Substituting values of x in (i),

we get : $y = \frac{5}{3}, \frac{1}{3}$

points on the curve are $\left(1, \frac{5}{3}\right)$ and $\left(-1, \frac{1}{3}\right)$

29	Here $f: N \to R$, $f(x) = 4x^2 + 12x + 15$ Let $x_1, x_2 \in N$ and $f(x_1) = f(x_2)$ Then, $4x_1^2 + 12x_1 + 15 = 4x_2^2 + 12x_2 + 15$ $\Rightarrow 4x_1^2 + 12x_1 = 4x_2^2 + 12x_2$ $\Rightarrow 4(x_1^2 - x_2^2) + 12(x_1 - x_2) = 0$ $\Rightarrow (x_1 - x_2)[4(x_1 + x_2) + 12] = 0$ As $x_1, x_2 \in N$ so, $4(x_1 + x_2) + 12 \neq 0$ $\therefore (x_1 - x_2) = 0$ $\Rightarrow x_1 = x_2$ so, f is one-one. - OR -
	$|x| = \begin{cases} x & ,x \ge 0 \\ -x & ,x < 0 \end{cases} \Rightarrow f(x) = \begin{cases} \frac{x}{1+x}, & x \ge 0 \\ \frac{x}{1-x}, & x < 0 \end{cases}$ **For x ≥ 0** $f(x_1) = \frac{x_1}{1+x_1}$ $f(x_2) = \frac{x_2}{1+x_2}$ $f(x_1) = f(x_2)$ $\frac{x_1}{1+x_1} = \frac{x_2}{1+x_2}$ $x_1(1+x_2) = x_2(1+x_1)$ $x_1 + x_1x_2 = x_2 + x_2x_1$ $x_1 = x_2$ **For x < 0** $f(x_1) = \frac{x_1}{1-x_1}$ $f(x_2) = \frac{x_2}{1-x_2}$ Putting $f(x_1) = f(x_2)$ $\frac{x_1}{1-x_1} = \frac{x_2}{1-x_2}$ $x_1(1-x_2) = x_2(1-x_1)$ $x_1 - x_1x_2 = x_2 - x_2x_1$ $x_1 = x_2$ Hence, if $f(x_1) = f(x_2)$, then $x_1 = x_2$ ∴ f is **one-one** **For x ≥ 0** $f(x) = \frac{x}{1+x}$ Let f(x) = y, $y = \frac{x}{1+x}$ $x = \frac{y}{1-y}$, for x ≥ 0 **For x < 0** $f(x) = \frac{x}{1-x}$ Let f(x) = y $y = \frac{x}{1-x}$ $x = \frac{y}{1+y}$, for x < 0 Here, $y \in \{x \in \mathbf{R}: -1 < x < 1\}$ So, x is defined for all values of y. ∴ f is **onto** Hence, f is one-one and onto.
30	$A = \begin{bmatrix} 2 & 4 & -6 \\ 7 & 3 & 5 \\ 1 & -2 & 4 \end{bmatrix}$ $A' = \begin{bmatrix} 2 & 7 & 1 \\ 4 & 3 & -2 \\ -6 & 5 & 4 \end{bmatrix}$ $P = \frac{1}{2}(A+A') = \frac{1}{2}\begin{bmatrix} 4 & 11 & -5 \\ 11 & 6 & 3 \\ -5 & 3 & 8 \end{bmatrix}$ $= \begin{bmatrix} 2 & \frac{11}{2} & -\frac{5}{2} \\ \frac{11}{2} & 3 & \frac{3}{2} \\ -\frac{5}{2} & \frac{3}{2} & 4 \end{bmatrix}$ Since $P' = P$ $\therefore$ P is a symmetric matrix. Let $Q = \frac{1}{2}(A-A') = \frac{1}{2}\begin{bmatrix} 0 & -3 & -7 \\ 3 & 0 & 7 \\ 7 & -7 & 0 \end{bmatrix}$ $= \begin{bmatrix} 0 & -\frac{3}{2} & -\frac{7}{2} \\ \frac{3}{2} & 0 & \frac{7}{2} \\ \frac{7}{2} & -\frac{7}{2} & 0 \end{bmatrix}$ $Q' = -\begin{bmatrix} 0 & -\frac{3}{2} & -\frac{7}{2} \\ \frac{3}{2} & 0 & \frac{7}{2} \\ \frac{7}{2} & -\frac{7}{2} & 0 \end{bmatrix} = -Q$ Since $Q' = -Q$ $\therefore$ Q is a skew symmetric matrix.
	Also $P + Q = \begin{bmatrix} 2 & \frac{11}{2} & -\frac{5}{2} \\ \frac{11}{2} & 3 & \frac{3}{2} \\ -\frac{5}{2} & \frac{3}{2} & 4 \end{bmatrix} + \begin{bmatrix} 0 & -\frac{3}{2} & -\frac{7}{2} \\ \frac{3}{2} & 0 & \frac{7}{2} \\ \frac{7}{2} & -\frac{7}{2} & 0 \end{bmatrix}$ $= \begin{bmatrix} 2 & 4 & -6 \\ 7 & 3 & 5 \\ 1 & -2 & 4 \end{bmatrix} = A$

31

Given that $y = 3\cos(\log x) + 4\sin(\log x)$

$$\frac{dy}{dx} = \frac{d}{dx}(3\cos(\log x) + 4\sin(\log x)) \quad = -3\sin(\log x).\frac{1}{x} + 4\cos(\log x).\frac{1}{x}$$

$$\Rightarrow x\frac{dy}{dx} = -3\sin(\log x) + 4\cos(\log x)$$

$$x\frac{d^2y}{dx^2} + \frac{dy}{dx}.\frac{d}{dx}x = \frac{d}{dx}[-3\sin(\log x) + 4\cos(\log x)]$$

$$= -3\cos(\log x).\frac{1}{x} - 4\sin(\log x).\frac{1}{x} \; = -\frac{1}{x}[3\cos(\log x) + 4\sin(\log x)] = -\frac{1}{x}.y$$

$$\Rightarrow x\frac{d^2y}{dx^2} + \frac{dy}{dx} = -\frac{1}{x}y \qquad \Rightarrow x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} = -y \qquad \Rightarrow x^2y_2 + xy_1 + y = 0$$

32

Given that $(a,b)R(c,d)$ iff $ad(b+c) = bc(a+$

$$\frac{b+c}{bc} = \frac{a+d}{ad} \Rightarrow \frac{1}{c} + \frac{1}{b} = \frac{1}{d} + \frac{1}{a}$$

$$\Rightarrow \frac{1}{a} - \frac{1}{b} = \frac{1}{c} - \frac{1}{d}$$

$(a,b)R(c,d)$ iff $\frac{1}{a} - \frac{1}{b} = \frac{1}{c} - \frac{1}{d}$

Reflexive:

$(a,b) \in N \times N \Rightarrow \frac{1}{a} - \frac{1}{b} = \frac{1}{a} - \frac{1}{b} \Rightarrow (a,b)R(a,b)$

$\therefore$ R is reflexive.

Symmetric:

$(a,b)R(c,d) \Rightarrow \frac{1}{a} - \frac{1}{b} = \frac{1}{c} - \frac{1}{d}$

$\Rightarrow \frac{1}{c} - \frac{1}{d} = \frac{1}{a} - \frac{1}{b} \Rightarrow (c,d)R(a,b)$

$\therefore$ R is symmetric

Transitivity:

$(a,b)R(c,d)$ and $(c,d)R(e,f)$

$\Rightarrow \frac{1}{a} - \frac{1}{b} = \frac{1}{c} - \frac{1}{d}$ and $\frac{1}{c} - \frac{1}{d} = \frac{1}{e} - \frac{1}{f}$

$\Rightarrow \frac{1}{a} - \frac{1}{b} = \frac{1}{e} - \frac{1}{f} \Rightarrow (a,b)R(c,d)$

$\therefore$ R is transitive.

R is reflexive, symmetric and transitive

R is an equivalence relation.

(i) R = {(a, b) : |a − b| is a multiple of 4}

Reflexivity:

Let $a \in A$, $(a,a) \in R \Rightarrow |a-a| = 0$ is a multiple of 4

$\Rightarrow$ R is reflexive.

Symmetricity:

Let $(a,b) \in R \Rightarrow |a-b|$ is a multiple of 4

$\Rightarrow |-(b-a|$ is a multiple of 4

$\Rightarrow |(b-a|$ is a multiple of 4

$\Rightarrow (b,a) \in R$

$\Rightarrow$ R is symmetric.

Transitivity

$(a,b),(b,c) \in R$

$\Rightarrow |(a-b|$ is a multiple of 4 and $|(b-c|$ is a multiple of 4

$\Rightarrow (a-b)$ is a multiple of 4 and $(b-c)$is a multiple of 4

$\Rightarrow (a-c) = (a-b) + (b-c)$ is a multiple of 4

$\Rightarrow |(a-c|$ is a multiple of 4

$\Rightarrow (a,c) \in R$

$\Rightarrow$ R is Transitive.

$\Rightarrow$ R is an equivalence relation.

$|(1-1| = 0$ is a multiple of 4

$|(5-1| = 4$ is a multiple of 4

$|(9-1| = 8$ is a multiple of 4

The set of elements related to 1 is {1,5,9}

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$$A=\begin{vmatrix}1 & 2 & 3\\ 2 & 3 & -3\\ -3 & 2 & -4\end{vmatrix}$$

$$|A|=1(-12+6)-2(-8-9)+3(4+9)$$
$$=-6+34+39=67\neq 0$$

$$\text{Adj. } A=\begin{bmatrix}-6 & 14 & -15\\ 17 & 5 & 9\\ 13 & -8 & -1\end{bmatrix}$$

$$A^{-1}=\frac{1}{67}\begin{bmatrix}-6 & 14 & -15\\ 17 & 5 & 9\\ 13 & -8 & -1\end{bmatrix}$$

The matrix form of the equations is

$$\begin{bmatrix}1 & 2 & -3\\ 2 & 3 & 2\\ 3 & -3 & -4\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix}=\begin{bmatrix}-4\\ 14\\ -15\end{bmatrix}$$

$$A'X=B$$
$$X=(A')^{-1}B$$
$$=(A^{-1})'B$$
$$=\frac{1}{67}\begin{bmatrix}-6 & 17 & 13\\ 14 & 5 & -8\\ -15 & 9 & -1\end{bmatrix}\begin{bmatrix}-4\\ 14\\ -15\end{bmatrix}$$
$$=\frac{1}{67}\begin{bmatrix}24+238-195\\ -56+70+120\\ 60+126+15\end{bmatrix}$$
$$=\frac{1}{67}\begin{bmatrix}67\\ 134\\ 201\end{bmatrix}=\begin{bmatrix}1\\ 2\\ 3\end{bmatrix}$$

$x=1,\quad y=2,\quad z=3$

34

$u=y^x$, $v=x^y$ and $w=x^x$,

we get $u+v+w=a^b$

$$\frac{du}{dx}+\frac{dv}{dx}+\frac{dw}{dx}=0$$

$$\log u=x\log y$$
$$\frac{du}{dx}=u\left(\frac{x}{y}\frac{dy}{dx}+\log y\right)$$
$$=y^x\left[\frac{x}{y}\frac{dy}{dx}+\log y\right]$$

$$\log v=y\log x$$
$$\frac{dv}{dx}=v\left[\frac{y}{x}+\log x\frac{dy}{dx}\right]$$
$$=x^y\left[\frac{y}{x}+\log x\frac{dy}{dx}\right]$$

$$\frac{dw}{dx}=w\,(1+\log x)$$
$$=x^x\,(1+\log x)\qquad \log w=x\log x$$

$$y^x\left(\frac{x}{y}\frac{dy}{dx}+\log y\right)+x^y\left(\frac{y}{x}+\log x\frac{dy}{dx}\right)+x^x\,(1+\log x)=0$$

$$(x\,.\,y^{x-1}+x^y\,.\,\log x)\,\frac{dy}{dx}=-\,x^x\,(1+\log x)-y\,.\,x^{y-1}-y^x\log y$$

$$\frac{dy}{dx}=\frac{-[y^x\log y+y\,.\,x^{y-1}+x^x(1+\log x)]}{x\,.\,y^{x-1}+x^y\log x}$$

35

Let $I=\int_{\frac{\pi}{6}}^{\frac{\pi}{3}}\frac{\sin x+\cos x}{\sqrt{\sin 2x}}dx$

$$\Rightarrow I=\int_{\frac{\pi}{6}}^{\frac{\pi}{3}}\frac{(\sin x+\cos x)}{\sqrt{-(-\sin 2x)}}dx$$

$$\Rightarrow I=\int_{\frac{\pi}{6}}^{\frac{\pi}{3}}\frac{\sin x+\cos x}{\sqrt{-(-1+1-2\sin x\cos x)}}dx$$

$$\Rightarrow I=\int_{\frac{\pi}{6}}^{\frac{\pi}{3}}\frac{(\sin x+\cos x)}{\sqrt{1-(\sin^2 x+\cos^2 x-2\sin x\cos x)}}dx$$

$$\Rightarrow I=\int_{\frac{\pi}{6}}^{\frac{\pi}{3}}\frac{(\sin x+\cos x)dx}{\sqrt{1-(\sin x-\cos x)^2}}$$

$$I=\int_{\frac{\pi}{6}}^{\frac{\pi}{3}}\frac{\sin x+\cos x}{\sqrt{\sin 2x}}dx\quad=\int_{\frac{1-\sqrt{3}}{2}}^{\frac{\sqrt{3}-1}{2}}\frac{dt}{\sqrt{1-t^2}}$$

$$I=\int_{-\left(\frac{\sqrt{3}-1}{2}\right)}^{\frac{\sqrt{3}-1}{2}}\frac{dt}{\sqrt{1-t^2}}$$

As $\frac{1}{\sqrt{1-(-t)^2}}=\frac{1}{\sqrt{1-t^2}}$,

hence, $\frac{1}{\sqrt{1-t^2}}$ is an even function

$$\Rightarrow I=2\int_0^{\frac{\sqrt{3}-1}{2}}\frac{dt}{\sqrt{1-t^2}}\quad=[2\sin^{-1}t]_0^{\frac{\sqrt{3}-1}{2}}$$

$$=2\sin^{-1}\left(\frac{\sqrt{3}-1}{2}\right)$$

$(\sin x - \cos x) = t \Rightarrow (\sin x + \cos x)dx = dt$

When $x = \frac{\pi}{6}, t = \left(\frac{1-\sqrt{3}}{2}\right)$,

when $x = \frac{\pi}{3}, t = \left(\frac{\sqrt{3}-1}{2}\right)$

- OR -

$\frac{3x+5}{x^3-x^2-x+1} = \frac{3x+5}{(x-1)^2(x+1)}$

$\frac{3x+5}{(x-1)^2(x+1)} = \frac{A}{(x-1)} + \frac{B}{(x-1)^2} + \frac{C}{(x+1)}$

$\Rightarrow 3x + 5 = A(x-1)(x+1) + B(x+1) + C(x-1)^2$

$\Rightarrow 3x + 5 = A(x^2-1) + B(x+1) + C(x^2+1-2x)$

Equating the coefficients of x^2, ⇨ $A + C = 0$

Equating the coefficients of x ⇨ $B - 2C = 3$

Equating the constant term, ⇨ $-A + B + C = 5$

On solving, we get

$B = 4 \quad A = -\frac{1}{2}$ and $C = \frac{1}{2}$

$\therefore \frac{3x+5}{(x-1)^2(x+1)} = \frac{-1}{2(x-1)} + \frac{4}{(x-1)^2} + \frac{1}{2(x+1)}$

$\Rightarrow \int \frac{3x+5}{(x-1)^2(x+1)}dx$

$= -\frac{1}{2}\int \frac{1}{x-1}dx + 4\int \frac{1}{(x-1)^2}dx + \frac{1}{2}\int \frac{1}{(x+1)}dx$

$= -\frac{1}{2}\log|x-1| + 4\left(\frac{-1}{x-1}\right) + \frac{1}{2}\log|x+1| + C$

$= \frac{1}{2}\log\left|\frac{x+1}{x-1}\right| - \frac{4}{x-1} + C$

36 (i) Let length, breadth and height of the tank are x, x and y respectively

According to the Question

$\therefore x^2y = 500 \Rightarrow y = \frac{500}{x^2}$

Surface Area $= S = x^2 + 4xy \quad = x^2 + 4x\left(\frac{500}{x^2}\right) = x^2 + \frac{2000}{x} \quad \Rightarrow \quad \frac{dS}{dx} = 2x - \frac{2000}{x^2}$

For maxima or minima, $\frac{dS}{dx} = 0 \Rightarrow 2x - \frac{2000}{x^2} = 0 \Rightarrow x = 10m$

$\frac{d^2S}{dx^2} = 2 + \frac{4000}{x^3}$ and $\left(\frac{d^2S}{dx^2}\right)_{at\,x=10} = 2 + \frac{4000}{(10)^3} > 0$

$\therefore$ Surface Area is minimum when $x = 10m$

$\therefore$ Minimum Surface Area $= 100 + \frac{2000}{10} = 300m^2$

(ii) If $x = 10m$ then $y = 5m$

and Volume of the tank $= x^2y = (10)^2(5) = 500m^3$

New Volume $= (2x)^2y = 4x^2y = 4(10)^2(5) = 2000m^3$

$\therefore$ Increase in Volume of the tank $= 2000 - 500 = 1500m^3$

$\therefore$ % Increase in Volume of the tank $= 300\%$

37 (i) $P(x) = -5x^2 + 125x + 37500$

$38250 = -5x^2 + 125x + 37500$

$= 5x^2 - 125x + 750 = 0$

$= (x-10)(x-15) = 0 \Rightarrow x = 10, 15.$ But $x \neq 10$

$\therefore x = 15$

(ii) Put $x = 2$ in $P(x)$

$P(x) = -5x^2 + 125x + 37500$

$P(2) = -5(4) + 125(2) + 37500$

$=$ ₹ 37730

<table>
<tr>
<td></td>
<td>(iii) $P(x) = -5x^2 + 125x + 37500$
$P'(x) = -10x + 125$
$P'(x) = 0 \Rightarrow -10x + 125 = 0 \Rightarrow x = 12.5$
$P''(x) = -10 < 0$ when $x = 12.5$
$\therefore$ P (x) is maximum when $x = 12.5$
Put x= 12.5 in P(x)
Maximum profit = ₹38281.25

OR</td>
<td>$P(x) = -5x^2 + 125x + 37500$
$P'(x) = -10x + 125$
Profit is strictly increasing where
$P'(x)>0$
$\Rightarrow -10x+125>0$
$\Rightarrow x<12.5$
Profit is strictly increasing for $x \in (0,12.5)$
Profit is strictly decreasing where
$P'(x)<0$
$\Rightarrow -10x+125<0$
$\Rightarrow -10x<-125$
$\Rightarrow x>12.5$
Profit is strictly decreasing for $x \in (12.5, \infty)$</td>
</tr>
<tr>
<td>38</td>
<td colspan="2">(i) $AX = B$, where $A = \begin{bmatrix} 1 & 1 & 1 \\ 4 & 3 & 2 \\ 6 & 2 & 3 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $B = \begin{bmatrix} 21 \\ 60 \\ 70 \end{bmatrix}$

(ii) $|A| = \begin{vmatrix} 1 & 1 & 1 \\ 4 & 3 & 2 \\ 6 & 2 & 3 \end{vmatrix} = 1(9-4) - 1(12-12) + 1(8-18)) = 5 - 0 - 10 = -5$

(iii) $A^{-1} = \frac{1}{|A|}.\text{adj}A = -\frac{1}{5}\begin{bmatrix} 5 & -1 & -1 \\ 0 & -3 & 2 \\ -10 & 4 & -1 \end{bmatrix}$ cost of verity c = 8 Rupees

- OR -

$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = -\frac{1}{5}\begin{bmatrix} 5 & -1 & -1 \\ 0 & -3 & 2 \\ -10 & 4 & -1 \end{bmatrix}\begin{bmatrix} 21 \\ 60 \\ 70 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = -\frac{1}{5}\begin{bmatrix} -25 \\ -40 \\ -40 \end{bmatrix} \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ 8 \\ 8 \end{bmatrix} \Rightarrow x = 5,\ y = 8,\ z = 8$

$\Rightarrow$ Cost of pen A = ₹ 5; cost of pen B = ₹ 8 and cost of pen C = ₹ 8</td>
</tr>
</table>